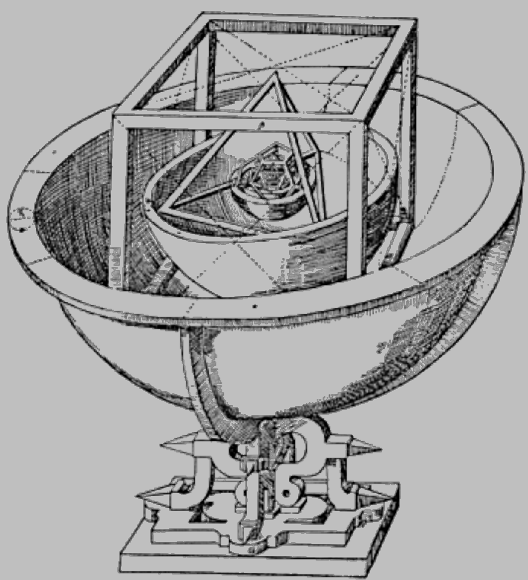
pst-kepler

The earth model of Johannes Kepler; v.0.01

January 1, 2026



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Package author(s):
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Contents

This package is dedicated to Jürgen Gilg (08.02.1966 – 06.05.2022). His work for PSTricks will never be forgotten.

Jean-François Burnol's tribute to Jürgen Gilg († 2022):

»It is with deep sadness that I learned of Jürgen's completely unexpected passing. How I regret not having visited him in Germany! I received his first email on January 2, 2018, some time after I began interacting with Thomas Söll. It was titled ›Polynomials with XINT‹ and is the direct inspiration for my TEX/LATEX package `polexpr`. Since then, Jürgen, Thomas, and I have had many exchanges about `xint`, `polexpr`, and also PS-Tricks, of which Jürgen was an expert, and where he was my teacher in everything. Jürgen, along with Thomas Söll and others, did extraordinary things with PS-Tricks. As the months and years went by, we talked more and more about other subjects, about events in our respective lives, and we shared painful experiences. I think Jürgen was incredibly kind-hearted; I feel like I've known him for ten or twenty years, but it's only been four …«

Jean-François Burnol, May 8, 2022

1 Curriculum Vitæ of Johannes Kepler (December 27, 1571 – November 15, 1630)

Johannes Kepler was born on December 27, 1571 in Weil (today: Weil der Stadt) as son of Katharina and Heinrich Kepler. He lived in the epoch of Renaissance till the beginning of the »Thirty Years' War«. The big change in spiritual subjects, which arised to that time, manifested itself in acrimonious religious wars during the reformation and intellectually for famous explorers to compete for a new true knowledge about the world.

Figure 1: Johannes Kepler

Nikolaus Kopernikus (1473-1543) described in his astronomical opus »*Von den Umdrehungen der Hemisphären*«, which he released short time before he died, that not the Earth, but the Sun is the center of the planet system, hence the Earth would be only a planet among other planets. Duke Christoph donated a scholarship for talented children for the University of Tübingen. In 1589 Kepler, who received such a scholarship, started to study in the subjects of Theology, Mathematics and Astronomy. In this connection he came to know of Kopernikus' revoluting thesis. After attending two convent schools Kepler came into the Tübinger monastery. Due to the fact of his critical intellect not fitting with all the dogmas of the post-lutherian Orthodoxy (amongst others Lord's Supper, geocentric world view), he didn't receive any employment in Württemberg anymore. So he had to go in a foreign country. In Graz he worked from 1594 to 1600 as a mathematician of the province. There he came into his own which was the astronomy. In 1597 he married Barbara Müller.

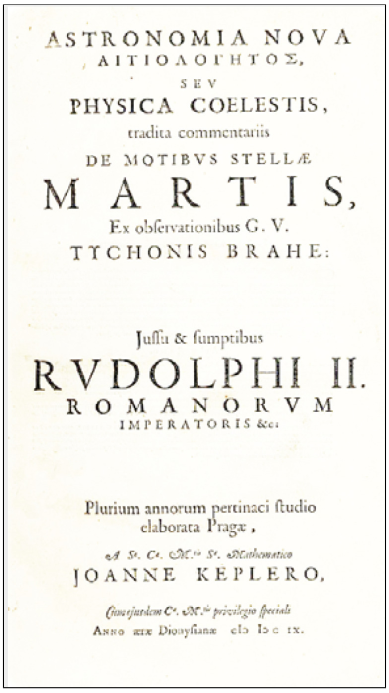
With 24, Kepler published his first opus »*Mysterium cosmographicum*« (»*The Mysteries of the Universe*«). [5] In 1600 he moved with his wife to Prague, to become a collaborator of Tycho Brahe. After Brahe's death in the following year, Kepler became his successor as an astronomer under Emperor Rudolf II. and as an imperial mathematician respectively and continued Brahe's astronomical work that he has left. In 1605 he realized, that the orbit of Mars is not a circle, but an ellipse. This finding is the base for the Kepler Laws.

Figure 2: Tycho Brahe

The first two were released in 1609 in »*Astronomia nova*« (»*New Astronomy*«). His Third Law Kepler posted in 1619 in the opus »HARMONICES MUNDI« (»*Harmony of the World*«).

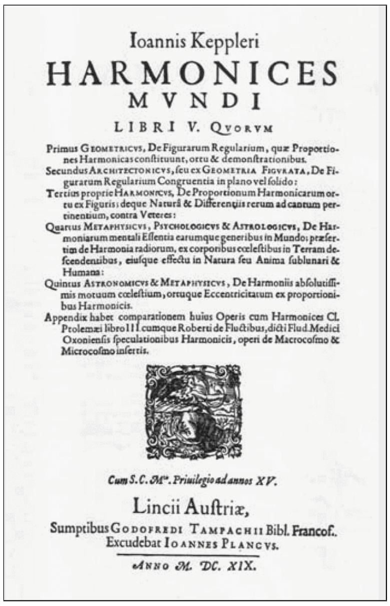
Apart from the astronomy, Kepler contributed significant things in the subject of Optics. In 1611 »*Dioptrice* (»*Dioptre*«) was published, where he progressed in the theory of lenses of a telescope (with two convex lenses). After his wife died in 1611 and later his benefactor Rudolf II. as well, he became mathematician of the province in Linz. There he married Susanne Reutinger.

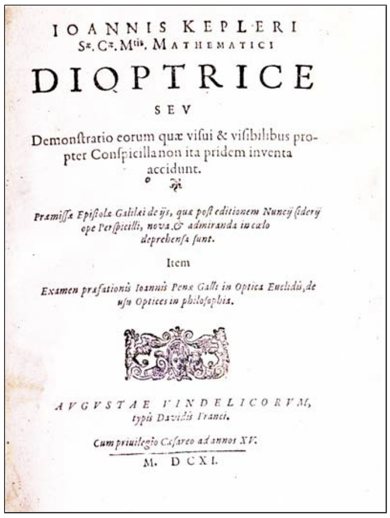
He published "*Abriss der kopernikanischen Astronomie*" (7 Volumes, 1618-22) and released the »*Tabulae Rudolphinae*« (»*Rudolphine Tables*«) in 1627. They include till


ASTRONOMIA NOVA
ΑΙΤΙΟΛΟΓΗΤΟΣ,
SEV
PHYSICA COELESTIS,
tradita commentariis
DE MOTIBVS STELLÆ
MARTIS,
Ex observationibus G. V.
TYCHONIS BRAHE:

Jussu & sumptibus
RVDOLPHI II.
ROMANORVM
IMPERATORIS &c:

Plurium annorum pertinaci studio
elaborata Pragæ,
A S^e. C^e. M^tis. S^e. Mathematico
JOANNE KEPLERO,
Cum ejusdem C^e. M^tis. privilegio speciali
Anno æræ Dionysianæ cIↄ Iↄ c IX.


Ioannis Keppleri
HARMONICES
MVNDI
LIBRI V. QVORVM
Primus GEOMETRICVS, De Figurarum Regularium, quæ Proportiones Harmonicas constituunt, ortu & demonstrationibus.
Secundus ARCHITECTONICVS, seu ex GEOMETRIA FIGVRATA, De Figurarum Regularium Congruentia in plano vel solido:
Tertius proprie HARMONICVS, De Proportionum Harmonicarum ortu ex Figuris; deque Naturâ & Differentiis rerum ad cantum pertinentium, contra Veteres:
Quartus METAPHYSICVS, PSYCHOLOGICVS & ASTROLOGICVS, De Harmoniarum mentali Essentia earumque generibus in Mundo; præsertim de Harmonia radiorum, ex corporibus cœlestibus in Terram descendentibus, eiusque effectu in Natura seu Anima sublunari & Humana:
Quintus ASTRONOMICVS & METAPHYSICVS, De Harmoniis absolutissimis motuum cœlestium, ortuque Eccentricitatum ex proportionibus Harmonicis.
Appendix habet comparationem huius Operis cum Harmonices Cl. Ptolemæi libro III. cumque Roberti de Fluctibus, dicti Flud. Medici Oxoniensis speculationibus Harmonicis, operi de Macrocosmo & Microcosmo insertis.
Cum S.C. M^tis. Privilegio ad annos XV.
Lincii Austriæ,
Sumptibus GODOFREDI TAMPACHII Bibl. Francof.
Excudebat IOANNES PLANCVS.
ANNO M. DC. XIX.


IOANNIS KEPLERI
S^e. C^e. M^tis. MATHEMATICI
DIOPTRICE
SEV
Demonstratio eorum quæ visui & visibilibus propter Conspicilla non ita pridem inventa accidunt.
Præmissæ Epistolæ Galilæi de iis, quæ post editionem Nuncij siderij ope Perspicilli, nova & admiranda in cœlo deprehensa sunt.
Item
Examen præfationis Ioannis Penæ Galli in Optica Euclidis, de usu Optices in philosophia.
AVGVSTAE VINDELICORVM,
typis Davidis Franci.
Cum privilegio Cæsareo ad annos XV.
M. DCXI.

then the most exact tables of the planet movements and for this reason with the appropriate planet positions.

From 1628 he was in attendance on Wallenstein in Ulm and Sagan. He mainly was asking fir some exact horoscopes. During that time the astronomy and astrology were strongly connected. At a later date it came out of its shadowy existence. An anecdote tells, that Kepler really prognosticates Wallenstein's date of death. He was well known for his precise horoscopes, although nowadays one can be doubtfully about that. For sure the astrology j ust was a means to an end for earning some money for his astronomical research.


TABULÆ
RUDOLPHINÆ,
A Phœnice illo Astronomorum
TYCHONE
IOANNES KEPLERUS,
ANNO M DC XXVII

In 1630 Kepler went on a journey to Linz. He fell ill in Regensburg and died on November 15, shotly after his arrival.

2 The symbolic of Kepler's solar system

With his debut feature, he likes to resolve, why God created exactly six and not any more planets. His first idea is to inscribe plain geometrical figures one within one to explain the number of planets. This failed, cause there are too many regular plain geometrical figures, but only five were needed.

Since the system of planets is three dimensional, Kepler exploits finally the **five regular Platonic Solids** for his thought experiment: the *Tetrahedron*, the *Hexahedron* (Cube), the *Octahedron*, the *Dodecahedron* and the *Icosahedron*. These regular solids are in— respectively circumscried by spheres. The ratio of the radiuses between inner and outer sphere expresses the ratio of the distances to the sun of adjacent planets (see figure 4 on page 6). For taking into account the orbit excentricity, Kepler emanates from a certain thickness of the spheres. The copernican data of distances suggest the

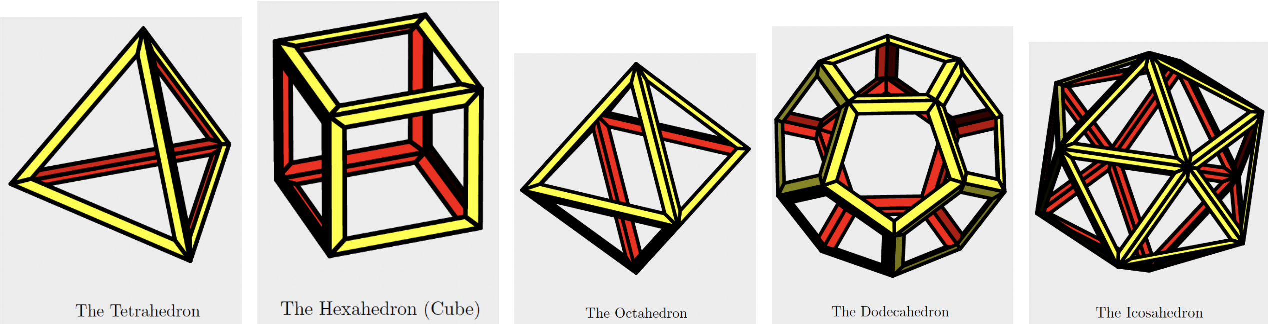
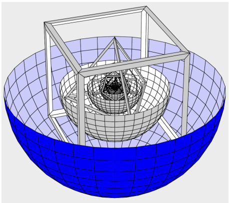
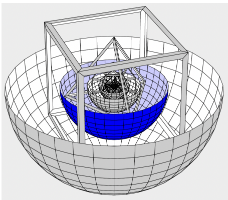
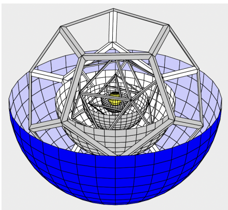
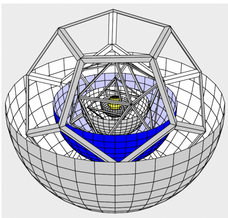
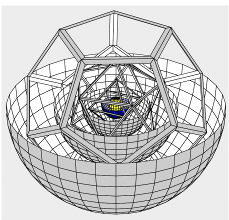


Figure 3: The five regular Platonic Solids

following order:

♄	Sphere of Saturn.		Hexahedron (Cube, the first of the regular bodies in geometry, showing the distance between the orbits of Saturn and Jupiter.
♃	Sphere of Jupiter.		Tetrahedron, or pyramid, touching the sphere of Jupiter on the outside and the sphere of Mars on the inside, and responsible for the greatest distance between the two planets.
	Sphere of Mars.		Dodecahedron, the third body, representing the distance extending from the sphere of Mars to the great orbit that encompasses the Earth and the Moon.
♁	Great orbit (of the Earth).		Icosahedron, which indicates the true distance between the great orbit and the sphere of Venus. Octahedron, which shows the distance between the sphere of Venus and that of Mercury.
☿	Sphere of Mercury.		Sun, the center of the stationary center of the Universe.

The greek philosopher Plato (about 300 B. C.) described detailed the solids and assigns them to the elements of the *Platonic World View*. They were assigned as schown in figure 5 on the next page.

In his dialog *Timaios*, Plato illustrates a model of the universe, wherein he combines the regular solids with the four elements Earth (Hexahedron/Cube), Water (Icosahedron), Fire (Tetrahedron) and Air (Octahedron). [3, 6] Plato associates the Dodecahedron with the *quinta essentia*, the Heavenly Ether. Every of the twelve side faces correspond to one of the twelve constellations.

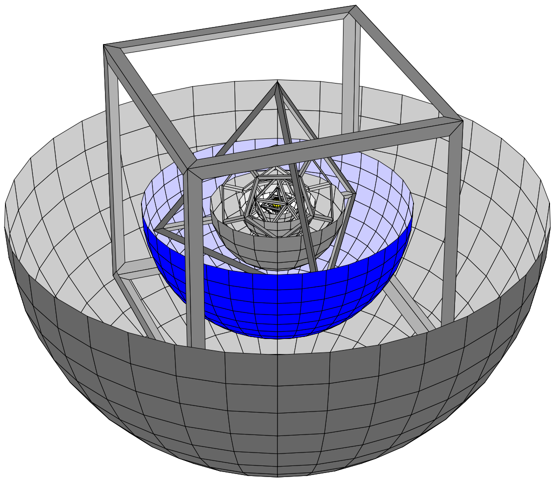


Figure 4: Kepler's solar system.

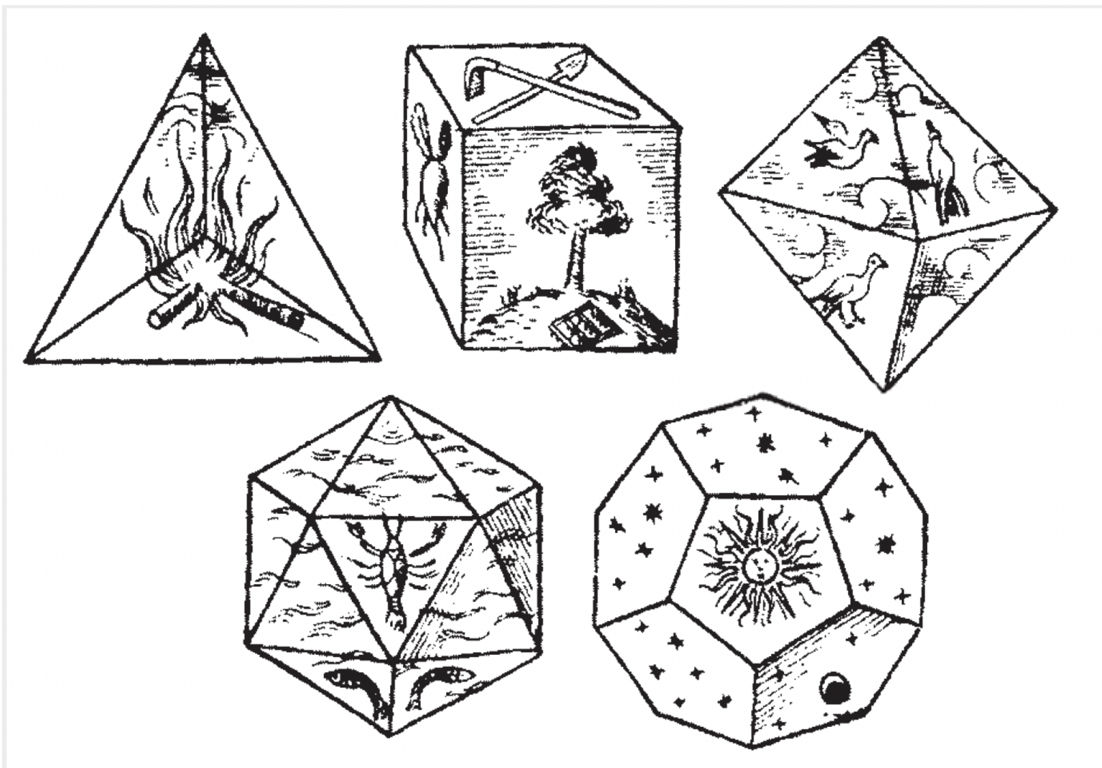


Figure 5: Platon's World View

3 Kepler's laws

3.1 Kepler's 1st Law

During his work in Prague, Kepler manifested his »First Law«. He phrased the following:

> »The cause is like so: The planet's orbit is not a circle; it comes in little by little on both sides and then back till the circumference of the circle in the perigree [the point of the moon's orbit that is closest to the Earth]. A such shaped orbit is called oval.«

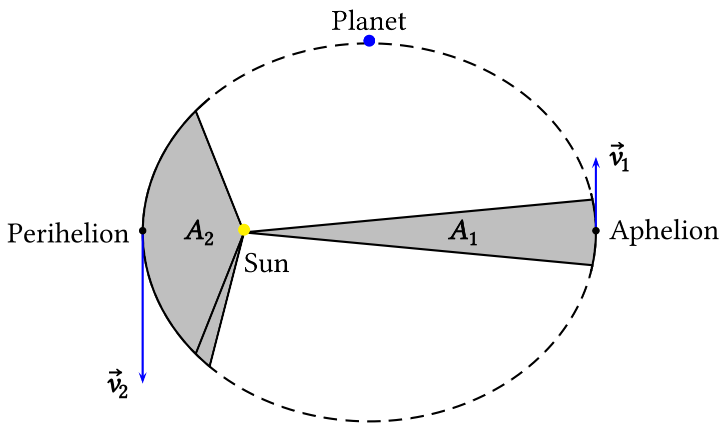
Nowadays it will be expressed:
All planets orbit the sun in elliptical orbits with the sun at one focus.

3.2 Kepler's 2nd Law

His »Second Law« reads like so:

> »Unvollkommenes, jedoch für die Sonnen- oder Erdbahn ausreichendes Verfahren zur Berechnung der Gleichungen auf Grund der physikalischen Hypothese. Da ich mir bewußt war, dass es unendlich viele Punkte auf dem Exzenter [außerhalb des Mittelpunktes liegende angebrachte Steuerungsscheibe] und entsprechend unendlich viele Abstände gibt, kam mir der Gedanke, dass in der Fläche des Exzenters alle diese Abstände enthalten seien.«

Nowadays it will be phrased like:



Perihelion is the point on its orbit when a planet is closest to the sun, *Aphelion* is the point on its orbit when a planet is farthest from the sun. The sun is in one focus of the elliptical orbit.

The line connecting a planet to the sun sweeps out equal areas in equal amounts of time. A planet moves faster nearer to the sun. Thus: $A_1 = A_2$.

3.3 Kepler's 3rd Law

Further researches leaded Kepler to his famous third Law of planet's movements, in 1619 published in »*Harmonices Mundi Libri V*«. [4]

»Allein es ist ganz sicher und stimmt vollkommen, dass die Proportion, die zwischen den Umlaufzeiten irgendzweier Planenten besteht, genau das Anderthalbe der Proportion der mittleren Abstände, d. h. der Bahnen selber, ist.«

In other words:
The squares of the periods T *of the planets are proportional t o the cubes of their semi-major axes* a.

Brought into a mathematical formula:

$$\frac{T^2}{a^3} = \text{const.}$$

The Third Law describes the complex relation between period and the distance to the sun.

The following table shows the relation:

- T measured as a multiple of an Earth year.
- a measured as a multiple of the distance Earth–Sun.

Planet	T	T^2	a	a^3
Mercury	0,241	0,058	0,387	0,058
Venus	0,615	0,378	0,723	0,378
Earth	1,000	1,000	1,000	1,000
Mars	1,881	3,538	1,524	3,540
Jupiter	11,860	140,660	5,203	140,852
Saturn	29,460	867,892	9,539	867,978

4 Background informations

4.1 John Banville

In his book »Kepler« (1981), John Banville imagines how the idea came to Kepler, pages 42 and 43 of the French edition, of which a German translation must exist. [1]

Perhaps he was mistaken, perhaps the universe was not an ordered construction governed by immutable laws? Who knows if God, like his creatures, does not ultimately prefer the temporal to the eternal, the near-perfect to the perfect, children's trumpets and the frenzied bleating of disorder to the music of the spheres? But, no, no, despite these doubts, no: his God was above all a God of order. The universe obeys geometric laws because geometry embodies the earthly paradigm of divine thought. He worked very late into the night, and went through the days swaying, in an ecstatic state. Summer came. He had been working non-stop for six months and had just reached the conclusion—if one could even call it a conclusion—that it was not so much the planets, their position and their speed, that he should concern himself with first, but the intervals between their orbits. If, for these distances, he possessed only Copernicus's estimates, which were scarcely more reliable than Ptolemy's, he nevertheless had to assume, for his own peace of mind, that they were precise enough to serve his purposes. Time and time again, he combined and recombined them in the hope of uncovering the relationship they concealed. Why were there only six planets? That was a question. But it was more pertinent to inquire into the distances that separated them. Why these distances and not others? He waited patiently for the rustling of wings.

On that July morning, though like so many other mornings, the angel appeared who brought him the answer. Johannes was in class. It was a hot and radiant day. A fly buzzed against the high window, a luminous diamond stood out on the floor at his feet. With glazed eyes, his students, petrified with boredom, stared at him without seeing him. Busy demonstrating a theorem of Euclid—though he tried afterward to remember which one, he could not—he had drawn an equilateral triangle on the board. He seized the heavy wooden compass and, immediately, as always, that monstrosity pricked him. He put his injured thumb to his mouth, resumed his work, and began to draw two circles; the first, inscribed inside the triangle, touched its three sides, while the second, circumscribed, encompassed it and passed through its three vertices. He stepped back to the lozenge of dusty light, blinked, and suddenly something, perhaps his heart, leaped, like an athlete performing a feat on a trampoline; for no reason, he said to himself, ecstatically: I will live forever. The proportion between the two circles was identical to that which existed between the orbits of Saturn and Jupiter, the most distant planets, and behold, between the circles that delimited this proportion was inscribed that basic figure which was the equilateral triangle in geometry. And if we placed a square between the orbits of Jupiter and Mars, a pentagon between Mars and Earth, and between Earth and Venus a … Yes. Oh, yes. The figure, the painting, even the walls of the room melted into a shimmering liquid, and the students of young Professor Kepler had the chance, a rare and gratifying spectacle, to see their master dab his eyes and hear him blow his nose noisily into a handkerchief of dubious whiteness.

4.2 Jacques Blamont

In his book »The Number and the Dream«, Jacques Blamont [2] describes his "discovery" as follows:

Now, Euclid showed that in three-dimensional space, only five solids (called Pythagorean) exist whose faces are identical: the tetrahedron (made up of four triangles), the cube (four squares), the octahedron (eight triangles), the dodecahedron (twelve pentagons), and the icosahedron (twenty triangles). Their symmetry allows them to be inscribed in a sphere (that is, their vertices are placed on it) or to be circumscribed about a sphere (that is, their sides are tangent to it). Five solids, five intervals between the planets! This coincidence provided the immediate solution to the enigma of the number of planets and the mystery of their distances; The radii of the spheres calculated by Copernicus allowed the five solids to be placed between the spheres in the following order:

Saturn-cube-Jupiter-tetrahedron-Mars-dodecahedron-Earth-icosahedron-Venus-octahedron-Mercury

The numbers had to be stretched a bit, but overall the agreement was good, except for Jupiter, »but no one will be surprised by that, given the great distance.« That's enough to make you forget many boils. The twenty-three-year-old leaped onto his pen and wrote a book which, despite its central error, was nonetheless a masterpiece, the Mysterium cosmographicum, whose genesis we know thanks to its preface:

> »I did not yet clearly see in what order the perfect solids should be arranged, and nevertheless I succeeded […] in arranging them so happily that later, when I checked these arrangements, I had nothing to change. I no longer regretted the lost time; I was no longer weary of my work; I shrank from no calculation, however difficult. Day and night, I performed my calculations to see if the proposition I had just formulated agreed with the orbits of Copernicus or if my joy would be carried away by the wind […]. In a few days, everything was in place. I saw the symmetrical solids fit one after the other with such precision between the appropriate orbits […] that if a peasant He asked, "On what hooks are the heavens fixed so as not to fall? It would be easy to answer.

5 The macros

5.1 World model

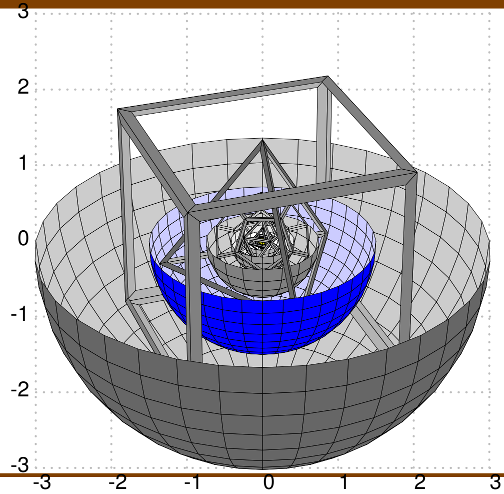
Show single Platonic solids

```
\psKeplerModel[Options](x,y)
```

The coordinates for the center are also optional, because they are preset to $(0,0)$.

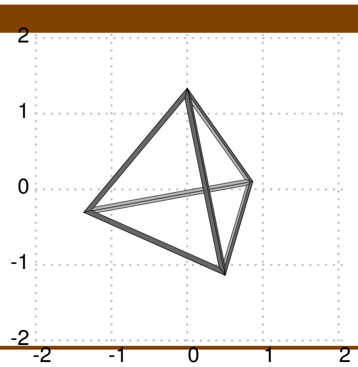
Kepler's world model

```
\begin{pspicture}[showgrid](-3,-3)(3,3)
\psKeplerModel[Decran=25]
\end{pspicture}
```



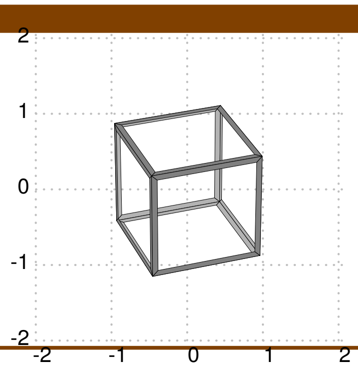
Tetrahedron Model

```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\psKeplerModel[type=tetrahedron,Decran=25]
\end{pspicture}
```



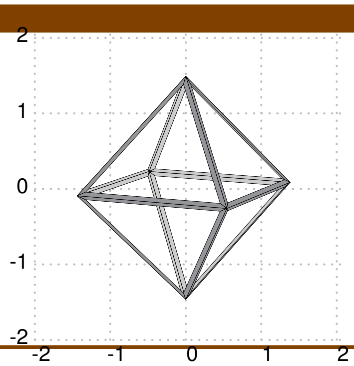
Hexahedron Model

```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\psKeplerModel[type=hexahedron,Decran=25]
\end{pspicture}
```



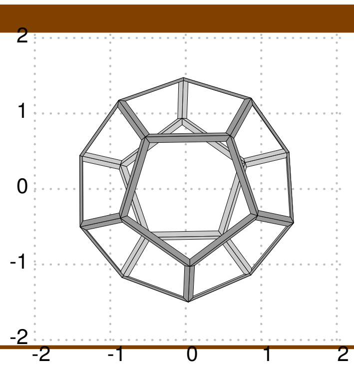
Octahedron Model

```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\psKeplerModel[type=octahedron,
  viewpoint=50 -20 10 rtp2xyz, Decran=25]
\end{pspicture}
```



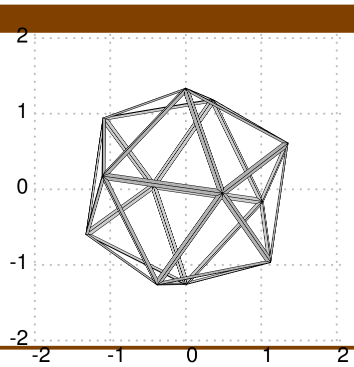
Dodekahedron Model

```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\psKeplerModel[type=dodekahedron,Decran=25]
\end{pspicture}
```



Icosahedron Model

```
\begin{pspicture}[showgrid](-2,-2)(2,2)
\psKeplerModel[type=icosahedron,Decran=25]
\end{pspicture}
```



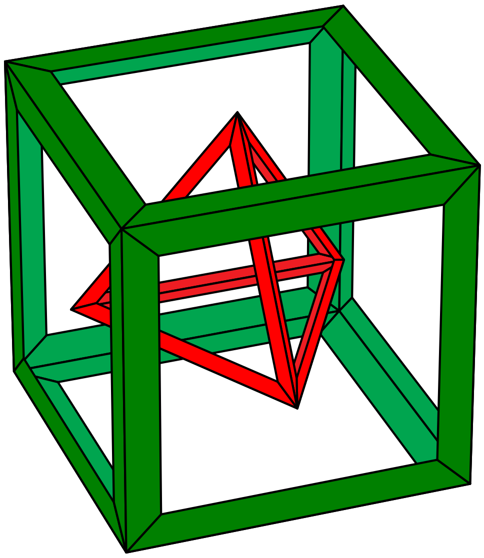
Show solids without a sphere

```
\psKeplerShow[Options](x,y)
```

The coordinates for the center are also optional, because they are preset to $(0,0)$. With this macro one can draw the five platonic solids. The scaling factor can be changed by the optional arguments `scaleTetra`, `scaleHexa`, `scaleOcta`, ... The values are preset to 2, 4, 6, ... All boolean options for the solids are preset to true. All five solids are shown if no optional argument is used.

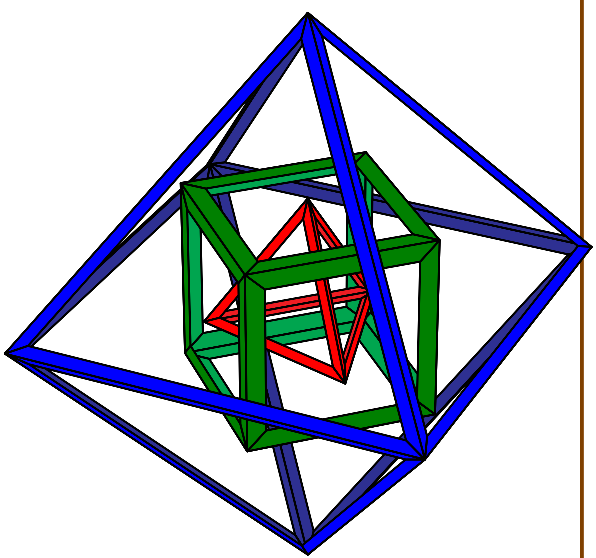
title

```
\begin{pspicture}(-4,-4)(4,4)
\psKeplerShow[unit=0.8,
              octahedron=false,
              dodecahedron=false,
              icosahedron=false]
\end{pspicture}
```



title

```
\begin{pspicture}(-5,-5)(5,5)
\psKeplerShow[unit=0.5,
              dodecahedron=false,
              icosahedron=false,
              scaleHexa=3.5,   % was preset to 4
              scaleOcta=5.5]  % was preset to 6
\end{pspicture}
```



5.2 2nd Law

The area computed by numerical integration[1] is evaluated to be $\Delta A_{1/2} = \frac{1}{2}\int_{\varphi_1}^{\varphi_2}(r(\varphi))^2 \mathrm{d}\varphi \approx 3.926\,990\,895\,678\,979$.

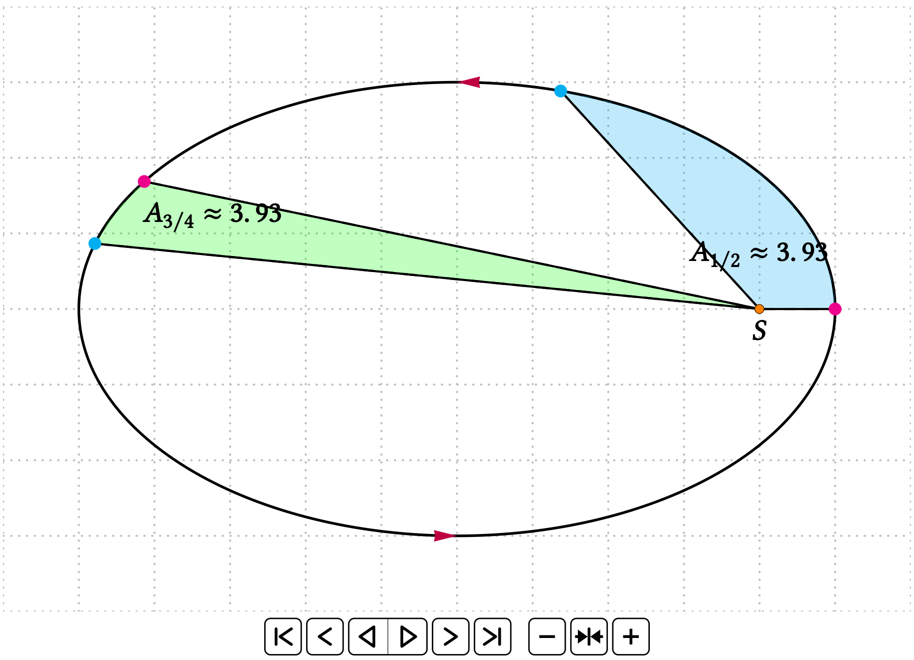
The exact value in this example is actually $\frac{1}{12}$ of the total ellipse area, i. e. $\frac{1}{12}\mathscr{A} \approx 3.926\,990\,816\,987\,243$.

The starting angle was $\varphi_1 = 0°$ and the ending angle $\varphi_2 = 132.335\,906\,455\,344\,7°$ and 7 evaluation points were used. With evaluation points separated by a bit less than $0.05°$ (115 evaluation points) we gain almost 6 digits of precision: $\Delta A_{1/2} \approx 3.926\,990\,816\,987\,757$. Let's use now starting angle $\varphi_1 = 0°$ and ending angle $\varphi_2 = 132.335\,906\,455\,344\,7°$ (itself computed approximately). We obtain (using 135 evaluations points) $\frac{1}{2}\int_{\varphi_1}^{\varphi_2}(r(\varphi))^2 \mathrm{d}\varphi \approx 3.926\,990\,895\,678\,979$. This is, not surprisingly, less precise that our previous evaluation on a shorter interval with comparable number of evaluation points.

The macro `\psKeplerAnimate` creates an inline PDF-animation which can only be viewed with the AdobeReader. This document also comes with a GIF-animation which can be used as standalone image or as part of a web-page.

Kepler's 2nd law as animation

```
\psKeplerAnimate % needs AdobeReader
```



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1 Simpson method with evaluations points separated by about **1** degrees

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